which may become sensible in modern refinements. In correcting the places of the stars, the aberration correction is made by the two terms—

$$A = k \cos \omega \cos \odot \qquad B = k \sin \odot$$

These depend on the longitude of the Sun relatively to the Earth, including all the perturbations. But they cannot contain any velocities common to Earth and Sun, such as, for instance, the minute rocking of the solar system by Jupiter. As Jupiter revolves, the whole solar system describes a minute orbit round the C.G., and there is a corresponding aberration. The amount is small, the coefficient being about $0''\cdot008$; but such a correction may be sensible in very refined investigations.

26. Again, there is a small term of about the same amount due to the rocking of the Earth by the Moon. I do not think it is included in the N.A. formulæ, although terms of smaller magnitude are retained—(*e.g.* in D we have the terms

$$+0''\cdot0008 \cos 2\Gamma' - 0''\cdot0027 \cos (3\odot - \Gamma). \text{ etc. etc.}).$$

Such terms only come into the observations of stars; planetary observations are fully corrected by the antedating method, if the time of transmission be correctly computed.

Note on the Calculation of Stellar Aberration.
By H. C. Plummer.

The following brief note is suggested by the concluding paragraphs of Professor Turner's paper on Aberration (p. 413), which I have been allowed to see in manuscript. He has pointed out that the usual method of calculating stellar aberration allows only partially for the effect of perturbations of the Earth's orbit. Where great accuracy is required, it may be well, at least as a check, to base the calculation directly on the use of the solar ephemeris.

The rectangular equatorial coordinates of the Sun are (X, Y, Z). Hence the components of the Earth's velocity relative to the Sun are $(-X', -Y', -Z')$, where X', Y', Z' can be derived from X, Y, Z by mechanical differentiation. Consider the rectangular directions (1) O S of the star; (2) O A perpendicular to O S, and such that the plane S O A contains the pole P; (3) O B perpendicular to both O S and O A. The components of the Earth's velocity vector in these directions are—

$$u = -\ (X' \cos \alpha \cos \delta + Y' \sin \alpha \cos \delta + Z' \sin \delta).$$
$$v = -\ (-X' \cos \alpha \sin \delta - Y' \sin \alpha \sin \delta + Z' \cos \delta).$$
$$w = -\ (-X' \sin \alpha + Y' \cos \alpha).$$

The first, u, is the reduction to the Sun of an observed radial

velocity.* The other two can be used for the calculation of the aberration in R.A. and declination. For, if U is the velocity of light, the aberration effects are (in circular measure)

$$\Delta\alpha . \cos\delta = w/U, \quad \Delta\delta = v/U.$$

Now the solar ephemeris gives X, Y, Z for 12-hourly intervals. Convenient units of length and time are therefore

mean distance of Earth to Sun

12 hours = 43200 mean solar seconds.

The ratio of these units divided by U is a number which expresses the light-equation θ in the unit of time chosen. Hence in seconds of arc

$$\Delta\alpha . \cos\delta = w\theta/\sin 1'', \quad \Delta\delta = v\theta/\sin 1'',$$

where

$$\theta/\sin 1'' = k\,T\cos\phi/\pi = [2{\cdot}06539]k,$$

k being the constant of aberration in seconds of arc, T one sidereal year in mean solar days, and $\sin\phi$ the eccentricity of the Earth's orbit. Hence

$$\Delta\alpha . \cos\delta = [2{\cdot}06539]\,k\,(X'\sin\alpha - Y'\cos\alpha).$$
$$\Delta\delta = [2{\cdot}06539]\,k\,(X'\cos\alpha\sin\delta + Y'\sin\alpha\sin\delta - Z'\cos\delta).$$

If we assume $k = 20''{\cdot}47$ we have

$$[2{\cdot}06539]\,k = [3{\cdot}37650] = 2379''{\cdot}6.$$

It is to be noticed that the aberration calculated in this way includes the effect of (1) the constant velocity vector perpendicular to the Earth's radius vector, (2) the constant velocity vector perpendicular to the line of apsides, and (3) the disturbance of the mean orbital motion of the Earth by the lunar and planetary perturbations; but it does not include the effect of (4) the motion of the Sun relative to the centre of mass of the solar system, which is very small and due virtually to the planets Jupiter and Saturn alone. And, of course, it is impossible to calculate the effect of that vector, constant or variable, which represents the motion of the solar system as a whole in space. The undisturbed elliptic motion of the Earth is represented by (1) and (2) together, but it is the practice deliberately to ignore the effect of (2), since it merely causes a small constant change in the mean place.† This fact has suggested a point for consideration in connection with photographic planetary observations, which was noticed and discussed by Mr. Cowell,‡ while as regards visual observations the same point had been considered by v. Oppolzer.§

* *Cf.* Schlesinger, *Astrophys. J.*, ix. p. 159.
† *Cf.* Bessel, *Tab. Reg.*, p. xix.
‡ *The Observatory*, xxiii. p. 447.
§ *A.N.*, 1560.

In order to remove the effect of the constant velocity parallel to the minor axis, and so to return to the usual practice, it is necessary to apply to $+X', +Y', +Z'$ the following constant corrections—

$$-{\cdot}0001414, \quad -{\cdot}0000257, \quad -{\cdot}0000112.$$

Also the effect of Jupiter in displacing the Sun relatively to the centre of mass of the solar system can be allowed for with sufficient accuracy by applying the corrections

$$-{\cdot}0000036 \sin l_J, \quad +{\cdot}0000033 \cos l_J, \quad +{\cdot}0000014 \cos l_J,$$

while the similar corrections for the action of Saturn are

$$-{\cdot}0000008 \sin l_S, \quad +{\cdot}0000007 \cos l_S, \quad +{\cdot}0000003 \cos l_S,$$

where l_J and l_S represent the heliocentric longitudes of Jupiter and Saturn.

If we compare the general formulæ for the aberration given above with the notation of the *Nautical Almanac* we see that

$$\begin{aligned} h \sin H = A \quad &= -\ [3{\cdot}37650]\ Y'. \\ h \cos H = B \quad &= +\ [3{\cdot}37650]\ X'. \\ i = A \tan \omega &= -\ [3{\cdot}37650]\ Z'. \end{aligned}$$

Hence the effect of all perturbations will be accounted for if in calculating A, B, and i (for Greenwich mean midnight) we use the formulæ:—

$$\begin{aligned} X' &= \left.\begin{array}{l}\Delta X - \tfrac{1}{2}\Delta^2 X + \tfrac{1}{3}\Delta^3 X \\ \quad -{\cdot}0001414 - {\cdot}0000036 \sin l_J - {\cdot}0000008 \sin l_S\end{array}\right\} \\ Y' &= \left.\begin{array}{l}\Delta Y - \tfrac{1}{2}\Delta^2 Y + \tfrac{1}{3}\Delta^3 Y \\ \quad -{\cdot}0000257 + {\cdot}0000033 \cos l_J + {\cdot}0000007 \cos l_S\end{array}\right\} \\ Z' &= \left.\begin{array}{l}\Delta Z - \tfrac{1}{2}\Delta^2 Z + \tfrac{1}{3}\Delta^3 Z \\ \quad -{\cdot}0000112 + {\cdot}0000014 \cos l_J + {\cdot}0000003 \cos l_S\end{array}\right\} \end{aligned}$$

The coordinate differences are to be formed directly from the 12-hourly solar ephemeris, beginning with the midnight of date.

As an example, let us take the date 1909 May 1, 12^h G.M.T. We find when the corrections have been applied

$$X' = -{\cdot}0056393, \quad Y' = +{\cdot}0059552, \quad Z' = +{\cdot}0025835,$$

which give

$$A = -[1{\cdot}15140], \quad B = -[1{\cdot}12773], \quad i = -[0{\cdot}78871],$$

while the *Nautical Almanac* gives

$$A = -[1{\cdot}15179], \quad B = -[1{\cdot}12762], \quad i = -[0{\cdot}7891].$$

If the former constants be applied to the star Regulus, the resulting effects of aberration are

$$\Delta\alpha.\cos\delta = +5''{\cdot}850, \quad \Delta\delta = -2''{\cdot}002,$$

whereas the result of using the *Nautical Almanac* data is to give

$$\Delta\alpha. \cos\delta = +\ 5''\cdot863, \quad \Delta\delta = -\ 2''\cdot008.$$

The differences are small, but quite sensible in comparison with some of the terms retained in calculating the nutation. There would thus seem to be a real advantage in calculating those of the day numbers which involve the aberration in the simple manner indicated above.

The question of the signs of the corrections is important, and a little intricate. That they have been rightly assigned may be seen thus. At the beginning of January the Sun passes perigee, and hence X′, Y′, Z′, of which the signs are all positive, are too large numerically. At the beginning of July the Sun passes apogee, and hence X′, Y′, Z′, of which the signs are all negative, are too small numerically. Hence the additive constant corrections required to bring the two sets together numerically must have negative signs.

Again, the position of a planet may be represented by

$$a\cos l, \quad a\sin l\cos i, \quad a\sin l\sin i,$$

and its velocity by

$$-\mathrm{V}\sin l, \quad \mathrm{V}\cos l\cos i, \quad \mathrm{V}\cos l\sin i.$$

The corresponding velocity induced in the Sun will be given by

$$m\mathrm{V}\sin l, \quad -m\mathrm{V}\cos l\cos i, \quad -m\mathrm{V}\cos l\sin i.$$

Now these components are to be added to those of the Earth relative to the Sun, and the latter are −X′, −Y′, −Z′. Hence as additive corrections to X′, Y′, Z′ they must be reversed in sign, and thus the signs given are verified.

Note on the observed time of minimum phase of an Algol Variable Star. By H. C. Plummer.

The idea of Dr. A. W. Roberts,* that the relative retardation of the minimum phase of an Algol variable could be used to determine the absolute dimensions of the orbit, was so ingenious that it seemed unfortunate when Father Stein † pointed out that the small interval of time in question must be interpreted differently. Both the dimensions of the orbit and the ratio of the masses are involved, and they cannot be separately determined from the photometric observations.

It may be worth pointing out that theoretically a knowledge of the instant of minimum phase, when used in conjunction with a spectrographic determination of the orbit of the bright component, will serve to determine both the dimensions of the orbit of the

* *M.N.*, lxvi. p. 123. † *M.N.*, lxviii. p. 490.